PARTICLE MOTION AND HEAT TRANSFER IN A PULSATING GAS STREAM

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Results are given of an analytical investigation of the motion of granular material and interphase heat transfer in an ascending two-phase stream with sinusoidal gas velocity fluctuations.

There is considerable interest in seeking ways of intensifying thermal processes (heating, drying, roasting) for granular materials in suspension. One of these ways is to impress pulsations on the gas stream velocity. The use of this principle has been suggested repeatedly [4, 6], but the laws of motion and heat transfer of granular material in a pulsating stream have not been sufficiently studied.

We know of only two papers [2, 3] devoted to the study of the motion and heat transfer of material particles in a gas stream with rectangular velocity pulsations. In addition, the influence was investigated in [1] of turbulent flow pulsations on the nature of the motion of weightless particles.

We shall examine the motion of particles of granular material in an ascending two-phase stream subjected to sinusoidal gas velocity pulsations:

$$w = w_m + w_a \sin \omega\tau.$$

For simplicity we shall make the following assumptions:

1) the particles are spherical, and their trajectories are parallel to the channel axis;

2) there are no collisions of the particles between themselves, nor with the channel walls, nor is there attenuation of the gas stream pulsations along the channel;

3) the aerodynamic resistance during washing of the particles by the stream, and the interphase heat transfer, are quasi-stationary [2];

4) the resistance coefficient does not depend on Re (the motion occurs in the self-similar region).

The usual equation of motion for particles of materials [4] in the case of a pulsating stream may be written in the form

$$\frac{du}{d\tau} g = \left(\frac{w_m + w_a \sin \omega\tau - u}{v_u}\right)^2 \times$$
$$\times \operatorname{sgn}(w_m + w_a \sin \omega\tau - u) - g, \qquad (1)$$

where the factor $\operatorname{sgn}(w_m + w_a \sin \omega\tau - u)$ takes into account the fact that the velocity of solid particles in a pulsating stream may exceed the gas velocity at particular times (then the aerodynamic resistance force slows the particles).

Nonlinear equations of this type do not integrate in quadratures. We shall seek an approximate solution of (1) for the quasi-stabilized section of the flow in the form [1]

$$u = u_m + u_a \sin(\omega\tau - \alpha). \qquad (2)$$

Replacing u and $du/d\tau$ in (1) by their values from (2), and going over to dimensionless velocities, we obtain

$$\bar{u}_a v_u \omega \sin(\omega\tau + \pi/2 - \alpha) =$$
$$= g\{(\bar{w}_m - \bar{u}_m) + [\bar{w}_a \sin \omega\tau - \bar{u}_a \sin(\omega\tau - \alpha)]\}^2 \times \qquad (3)$$
$$\times \operatorname{sgn}\{(\bar{w}_m - \bar{u}_m) + [\bar{w}_a \sin \omega\tau - \bar{u}_a \sin(\omega\tau - \alpha)]\} - g.$$

The slip velocity is evidently also a sinusoidal quantity

$$\bar{v} = \bar{v}_m + \bar{v}_a \sin(\omega\tau + \beta), \qquad (4)$$

where, on the basis of the known relations [7]

$$\bar{v}_a^2 = \bar{w}_a^2 + \bar{u}_a^2 - 2\bar{w}_a \bar{u}_a \cos\alpha, \qquad (5)$$

$$\operatorname{tg}\beta = \bar{u}_a \sin\alpha/(\bar{w}_a - \bar{u}_a \cos\alpha). \qquad (6)$$

Moreover, it is evident that

$$\bar{v}_m = \bar{w}_m - \bar{u}_m. \qquad (7)$$

Taking (4) and (7) into account, we obtain, in place of (3)

$$\bar{u}_a v_u \omega \sin(\omega\tau + \pi/2 - \alpha) = g[\bar{v}_m + \bar{v}_a \sin(\omega\tau + \beta)]^2 \times$$
$$\times \operatorname{sgn}[\bar{v}_m + \bar{v}_a \sin(\omega\tau + \beta)] - g. \qquad (8)$$

It is evident that (2) may serve as an approximate solution of (1), if the left and right sides of (8) coincide: a) in phase, b) in average value over a period, c) in amplitude.

Condition a) is satisfied if

$$\alpha + \beta = \pi/2. \qquad (9)$$

Solving the system of equations (6), (9), we obtain

$$\cos\alpha = \bar{u}_a/\bar{w}_a. \qquad (10)$$

Taking account of (10), (5) may be put in the form

$$\bar{v}_a^2 = \bar{w}_a^2 - \bar{u}_a^2. \qquad (11)$$

In the quasi-stabilized section of the stream, the mean value of particle acceleration over one period is zero. The left side of (8) satisfies this condition for any values inserted in it.

Therefore, condition (6) is equivalent to equating the mean value of the right side of (8) to zero. Then two cases are possible: $\bar{v}_a \leq \bar{v}_m$ and $\bar{v}_a > \bar{v}_m$.

With $v_a \leqslant \bar{v}_m$ the value of sgn $[\bar{v}_m + \bar{v}_a \sin(\omega\tau + \beta)]$ is positive; then, after integration and simple transformations, we obtain

$$2(\bar{v}_m^2 - 1) + \bar{v}_a^2 = 0. \quad (12)$$

If we put $\bar{v}_m = \bar{v}_a$ in (12), we find $\bar{v}_m = \bar{v}_a = \sqrt{2/3} \approx 0.817$. Therefore, (12) is valid for values $0.817 \leqslant \bar{v}_m \leqslant 1$. When $\bar{v}_m < 0.817$ (i.e., $\bar{v}_a > \bar{v}_m$), we obtain, in a similar fashion to the foregoing,

$$(2\bar{v}_m^2 + \bar{v}_a^2)\arcsin\frac{\bar{v}_m}{\bar{v}_a} + 3\bar{v}_m\bar{v}_a\sqrt{1 - \frac{\bar{v}_m^2}{\bar{v}_a^2}} - \pi = 0. \quad (13)$$

It follows from (12) and (13) that in the range $0 < \bar{v}_m \leq 1$, there exists between $\bar{v}_m$ and $\bar{v}_a$ a unique relation which is invariant with respect to the upward velocity of the particles and to the characteristics of the gas stream.

This relation is shown in Fig. 1. We find, typically, that in the quasi-stabilized section, the mean slip velocity ($v_m = \bar{v}_m v_u$) may be considerably lower than the upward velocity of the particles, but then its amplitude must be large enough. Therefore, transport of large particles of material is possible in a pulsating stream with smaller mean velocity and gas flow rate than in a steady stream.

A convenient result for a stream with a rectangular pulsation was obtained previously in [2, 3]. Evidently, this derivation may be extended to pulsations of any shape.

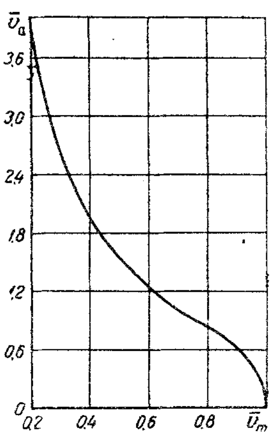

Fig. 1. Dependence of slip velocity amplitude $\bar{v}_a$ (m/sec) on its mean value $\bar{v}_m$ (m/sec).

Analysis of condition c) leads to the following result:

$$g(\bar{v}_a^2 + \bar{v}_m^2) = \bar{u}_a v_u \omega \quad (\bar{v}_m < 0.817); \quad (14)$$

$$2g\bar{v}_a\bar{v}_m = \bar{u}_a v_u \omega \quad (0.817 \leqslant \bar{v}_m \leqslant 1). \quad (15)$$

The system of six equations obtained, (7), (9)–(15), with 9 variables ($\bar{w}_m$, $\bar{w}_a$, $\bar{v}_m$, $\bar{v}_a$, $\bar{u}_m$, $\bar{u}_a$, ω, α, β) allows calculation of the remainder when three are known.

For example, from the given law of variation of gas stream velocity ($\bar{w}_m$, $\bar{w}_a$, ω) we may obtain the laws of particle motion and of slip velocity variation in the quasi-stabilized section.

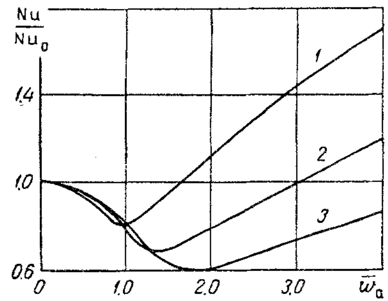

Fig. 2. Dependence of interphase heat transfer intensity Nu/Nu_0 in a pulsating stream on the gas velocity amplitude $\bar{w}_a$ (m/sec): 1) for the quasi-stabilized section of the stream; 2 and 3) for the section near the inlet, k = 1.8 and 3.

Calculation shows that in the region $\omega > 10\ \text{sec}^{-1}$, $\bar{v}_m > 0.2\ \bar{v}_a \approx \bar{w}_a$, the difference between the two quantities does not exceed 3–5%, and decreases with increase of pulsation frequency. Therefore, the curve of Fig. 1 also represents the relation $\bar{v}_m = f(w_a)$ with good enough accuracy.

It follows from (11)–(15) that the slip velocity does not depend on the mean gas velocity.

We shall examine the influence of pulsations on interphase heat transfer in an ascending two-phase stream. If the length of the stream is constant, then the amount of heat transferred is proportional to $Nu\,\tau_d$, where τ_d is the dwell time of the particles of material in the stream. The Nusselt number is a function of the Reynolds number [5]:

$$Nu = 2 + 0.16\,Re^{2/3}. \quad (16)$$

Since Re is proportional to $|v|$, and the second term on the right of (16) is considerably larger than the first, we may assume, with a known degree of accuracy, that

$$Nu \sim v^{2/3}, \quad Nu_{av}/Nu_0 = (v^{2/3})_{av}/v_u^{2/3} = (\bar{v}^{2/3})_{av},$$

or, taking account of (4),

$$\frac{Nu_{av}}{Nu_0} = \frac{1}{T}\int_0^T [\bar{v}_m + \bar{v}_a \sin(\omega\tau + \beta)]^{2/3}\, d\tau. \quad (17)$$

It follows from (17) that the ratio Nu_{av}/Nu_0 for the quasi-stabilized section of a pulsating stream is determined by the slip velocity, and does not depend on the frequency of pulsation. However, the present investigation does not take into account the acoustic properties of the gas column in the channel, and in real conditions, evidently, we may expect the pulsation frequency to have a definite influence on heat transfer intensity.

Since the indefinite integral corresponding to the right side of (17) cannot be expressed either in elementary or in elliptic functions, the value of Nu_{av}/Nu_0 for a series of values of $\overline{w}_a$ has been calculated by Simpson's rule. The results are given in Fig. 2. When $\overline{w}_a < 1.63$, the heat transfer intensity in the pulsating gas stream proves to be less than in a steady one, which is due to the decrease in mean slip velocity in the pulsating stream in comparison with the upward velocity of the particle. Appreciable enhancement of heat transfer may be obtained only at high values of $\overline{w}_a$.

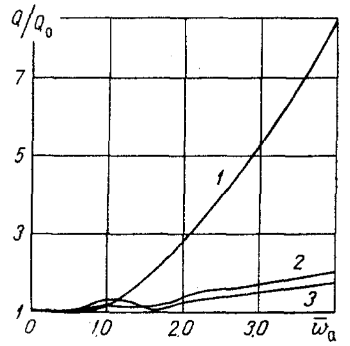

Fig. 3. Dependence of the amount of heat transferred in a pulsating stream, Q/Q_0, on the gas velocity amplitude $\overline{w}_a$ (m/sec): 1) for the quasi-stabilized section of the stream; 2 and 3) for the section near the inlet, k = 1.8 and 3.

The safety factor for pneumatic transportation of large particles of material is usually chosen in the range $k = w_0/v_u = 1.8–3.0$. We shall therefore consider that in the stabilized section of a steady stream the particle velocity is $u_0 = (0.8–2)v_u$, or, in dimensionless quantities, $\overline{u}_0 = 0.8–2$.

If, in a pulsating stream, we take the safety factor in terms of the mass flow rate of transporting agent $k = w_m/v_m$ within the same limits, then, allowing for (7), we obtain $\overline{u}_m = (0.8–2)\overline{v}_m$. Then, for identical values of k

$$\tau_d/\tau_{d\cdot 0} = 1/\overline{v}_m,$$

and the amounts of heat transferred, for the pulsating and the steady streams, are related by

$$\frac{Q}{Q_0} = \frac{Nu_{av}}{Nu_0}\frac{1}{\overline{v}_m}. \qquad (18)$$

It may be seen from Fig. 3 that the use of pulsations proves effective with $\overline{w}_a > 1$, the quantity Q/Q_0 rapidly increasing with increase of $\overline{w}_a$. From this point of view larger values of gas velocity amplitude should be chosen, but with $\overline{w}_a > \overline{w}_m$ the gas stream velocity must change sign periodically. Since the achievement of similar regimes encounters great technical difficulties, we shall restrict the examination only to those regimes for which

$$\overline{w}_a \leqslant \overline{w}_m. \qquad (19)$$

The maximum values of $\overline{w}_a$ satisfying condition (19) may evidently be determined with ease from Fig. 1 as the ordinates of points of intersection of the curve $w_a = f(v_m)$ with the lines $\overline{w}_m = k\overline{v}_m$. The values of $\overline{w}_a$ and corresponding values of Q/Q_0 are

k	1.8	2	2.5	3
w_a	1.16	1.23	1.39	1.54
Q/Q_0	1.32	1.42	1.66	1.90

For the accelerating section of a pulsating stream, there are considerable difficulties in obtaining even an approximate solution of the equation of motion of the particles of material, similar to (2). For a preliminary estimate of the heat-transfer intensity in the accelerating section we shall calculate the value Q/Q_0 for a small section ΔL of the stream near the initial section (the length of this section must be small compared to the scale of variation of characteristics of the stream).

Assuming that at the section in question the velocity of the material is close to zero, and reasoning in a way similar to the foregoing, we obtain

$$\frac{Nu}{Nu_0} = \frac{1}{\overline{w}_0^{2/3}}\frac{1}{T}\int_0^T (\overline{w}_m + \overline{w}_a \sin\omega\tau)^{2/3}\, d\tau. \qquad (20)$$

Relation (20) is shown in Fig. 2 for safety factor values of k = 1.8 and k = 3.

Regarding the motion of particles in the section ΔL as subject to uniform acceleration (with u = 0 at the inlet section), we obtain the following expression for the particle dwell time in section ΔL:

$$\frac{\tau_d}{\tau_{d\cdot 0}} = \sqrt{\left(\frac{du}{d\tau}\right)_0 \Big/ \left(\frac{du}{d\tau}\right)_{av}}. \qquad (21)$$

For a steady stream

$$(du/d\tau)_0 = g(\overline{w}_0^2 - 1) = (k^2 - 1)g,$$

and for a pulsating one

$$\left(\frac{du}{d\tau}\right)_{av} = \frac{g}{T}\int_0^T [(\overline{w}_m + \overline{w}_a \sin\omega\tau)^2 \operatorname{sgn}(\overline{w}_m + \overline{w}_a \sin\omega\tau) - 1]\, d\tau.$$

The ratio of the amounts of heat transferred in the section ΔL of the pulsating and steady streams has been calculated in the same way as for the quasi-stabilized section. The relation $Q/Q_0 = f(\overline{w}_a)$ for the section ΔL is shown in Fig. 3. When $\overline{w}_a > 1$ the influence of the pulsations is considerably stronger for the quasi-stabilized section than for the accelerating one. For values of $\overline{w}_a$ satisfying condition (19), the use of pulsations gives practically no perceptible result for the accelerating section of the stream.

NOTATION

g – acceleration due to gravity, m/sec^2; I – period of the gas stream velocity pulsations, sec; u – velocity of particles of disperse material, m/sec; v – slip velocity in the quasi-stabilized section of the stream, m/sec; v_u – upward velocity of the particles, m/sec; w – gas velocity, m/sec; τ – time, sec; ω – cyclic frequency, sec^{-1}; $\overline{u} = u/v_u$; $\overline{v} = v/v_u$; $\overline{w} = w/v_u$. Subscripts: a – amplitude of oscillations of a variable; m – mean value of a sinusoidal quantity; av – mean value of a variable sinusoidal quantity; 0 – refers to the steady gas stream (without pulsations).

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